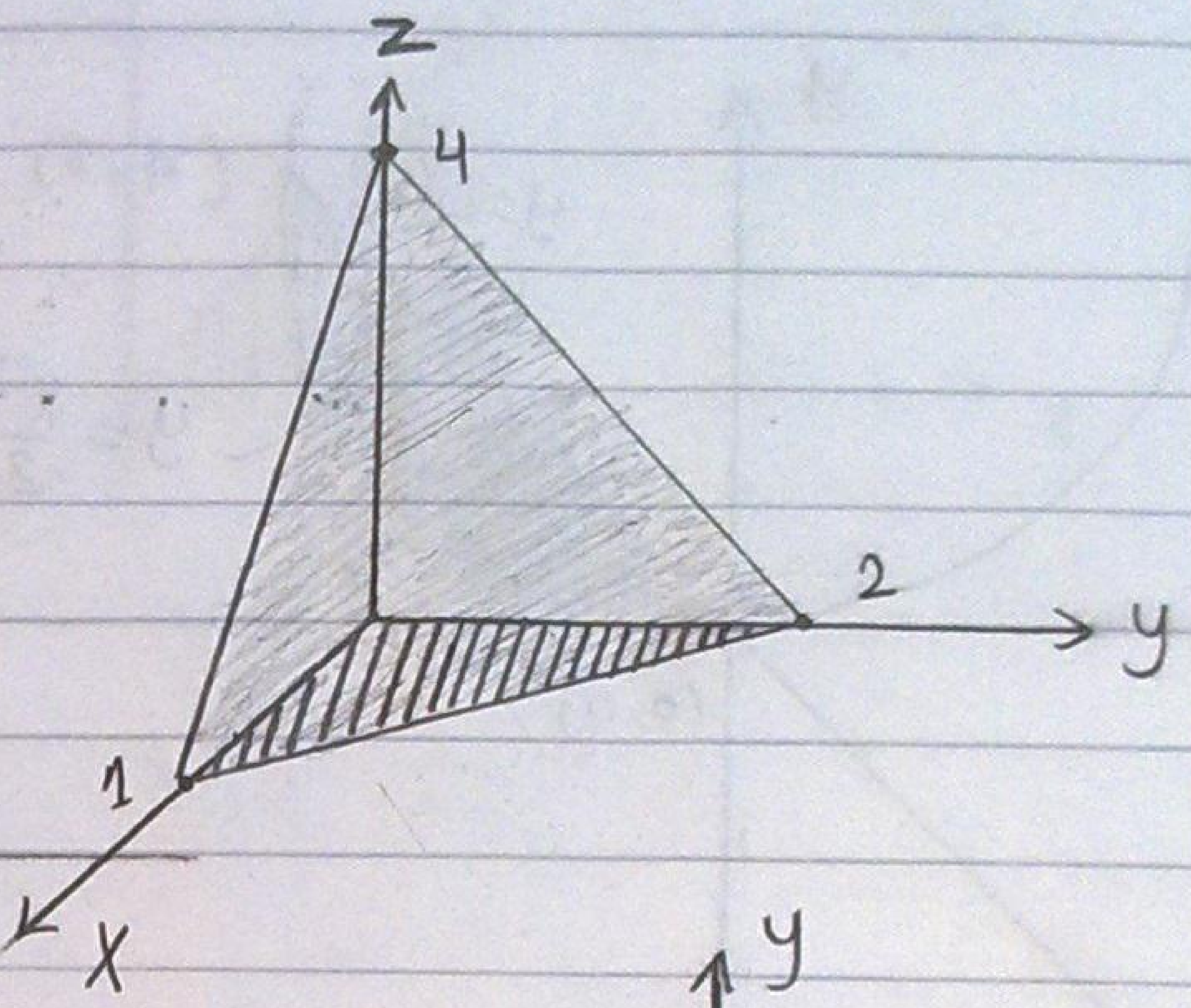
Ex:

Use the Integration to find the Volume of the tetrahedron هرم ثلاثي bounded by the Co-ordinates planes and the plane $Z = 4 - 4X - 2y$



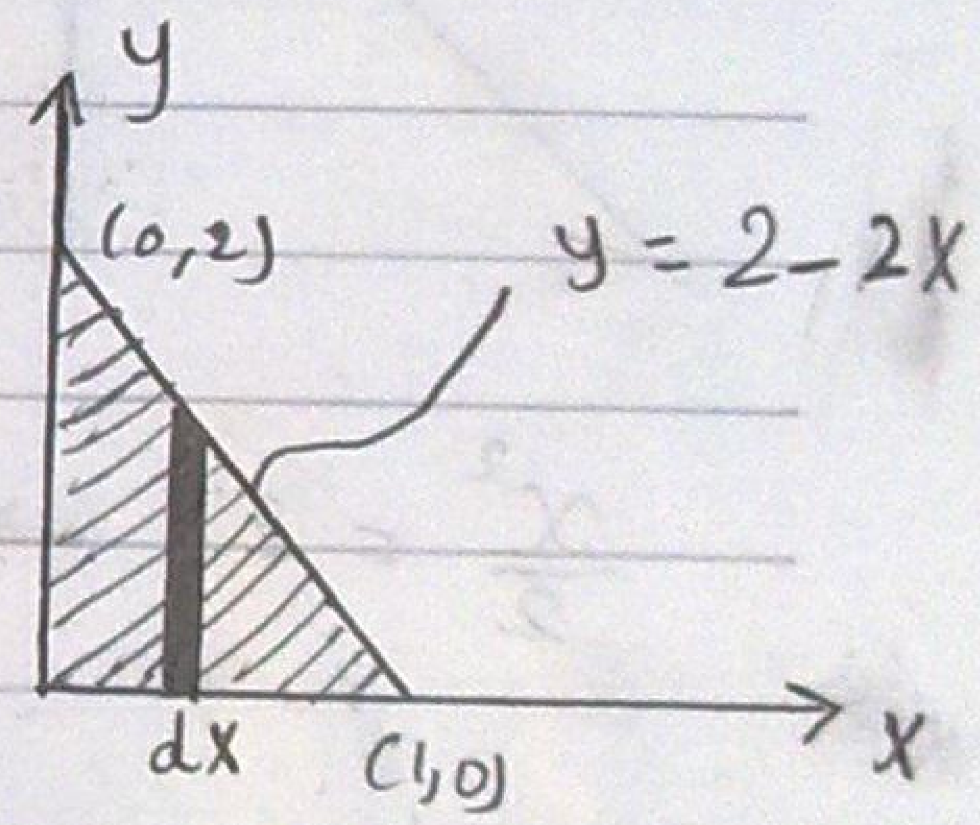
التقاطع مع المحاور

$Z, y = 0 \Rightarrow x = 1$

$Z, X = 0 \Rightarrow y = 2$

$X, y = 0 \Rightarrow Z = 4$

معادلة المستقيم = تقاطع المستوى الأفقي مع المحاور



$$\text{Volume} = \int_0^1 \int_0^{2-2X} (4 - 4X - 2y)\, dy\, dx$$

$$= \int_0^1 \left(4y - 4xy - y^2\right)_0^{2-2X} dx$$

$$= \int_0^1 \left(4(2-2X) - 4X(2-2X) - (2-2X)^2\right) dx$$

$$= \int_0^1 \left(8 - 8X - \cancel{8X} + 8X^2 - 4 - 4X^2 + \cancel{8X}\right) dx$$

$$= \int_0^1 (4 - 8X + 4X^2)\, dx = \left[4X - 4X^2 + \frac{4}{3}X^3\right]_0^1$$

$$= 4 - 4 + \frac{4}{3} = \boxed{\frac{4}{3}}$$

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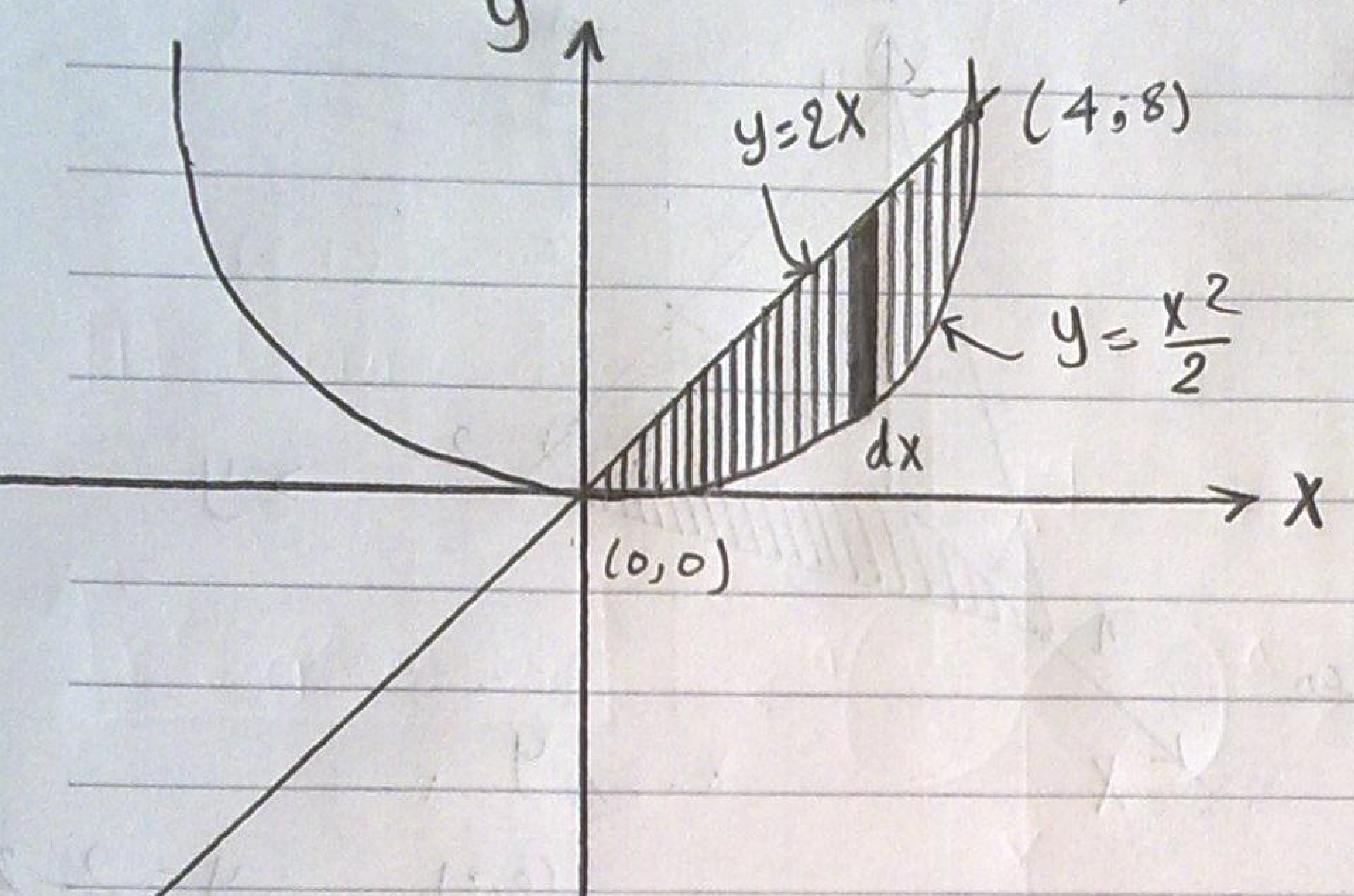
Use adouble integral to find the area of the Region R enclosed between the Parabola

$y = \frac{x^2}{2}$ and the line $y = 2x$



$$\iint_R f(x,y)\, dA = \text{Volume}$$

$$\iint (1)\, dA = \text{Area}$$

نقطة التقاطع

$\frac{x^2}{2} = 2x \Rightarrow x^2 = 4x \Rightarrow x(x-4) = 0$

$\Rightarrow x = 0 \quad \& \quad x = 4$

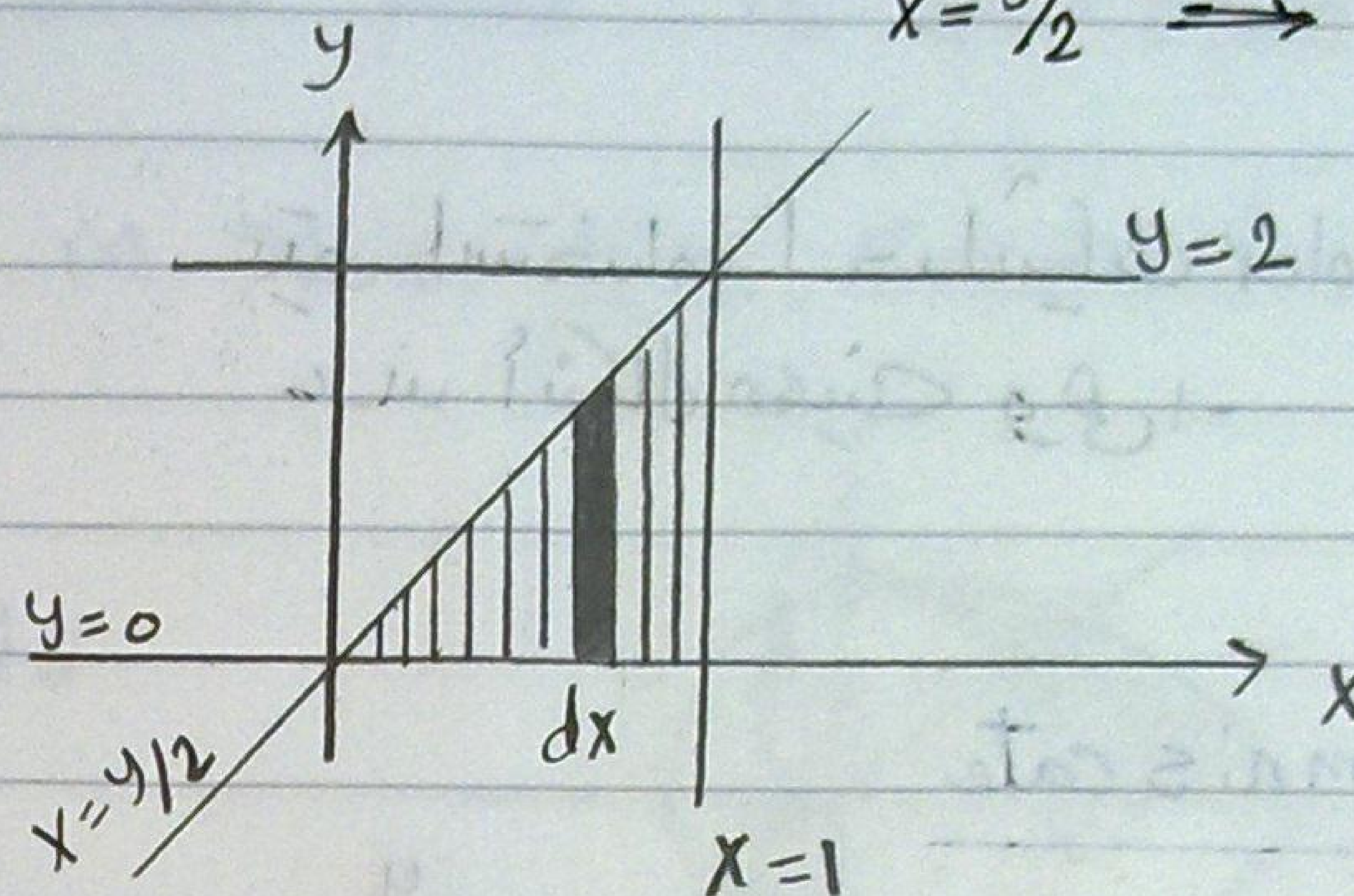
$y = 0 \quad \& \quad y = 8$

$$\text{Area} = \int_0^4 \int_{\frac{x^2}{2}}^{2x} dy \cdot dx = \int_0^4 [y]_{x^2/2}^{2x}\, dx = \int_0^4 \left(2x - \frac{x^2}{2}\right) dx$$

$$= \left[x^2 - \frac{x^3}{6}\right]_0^4 = 4^2 - \frac{(4)^3}{6} = \boxed{\frac{16}{3}}$$

Ex: solve $\int_0^2 \int_{y/2}^1 e^{x^2}\,dx\,dy$

$y=2$
$y=0$
$x=1$
$x=y/2 \longrightarrow y=2x$



$= \int_0^1 \int_0^{2x} e^{x^2}\,dy\,dx$

$= \int_0^1 \left[e^{x^2} y\right]_0^{2x} dx$

$= \int_0^1 2x e^{x^2}\,dx =$

$= \left[e^{x^2}\right]_0^1 = e-1$

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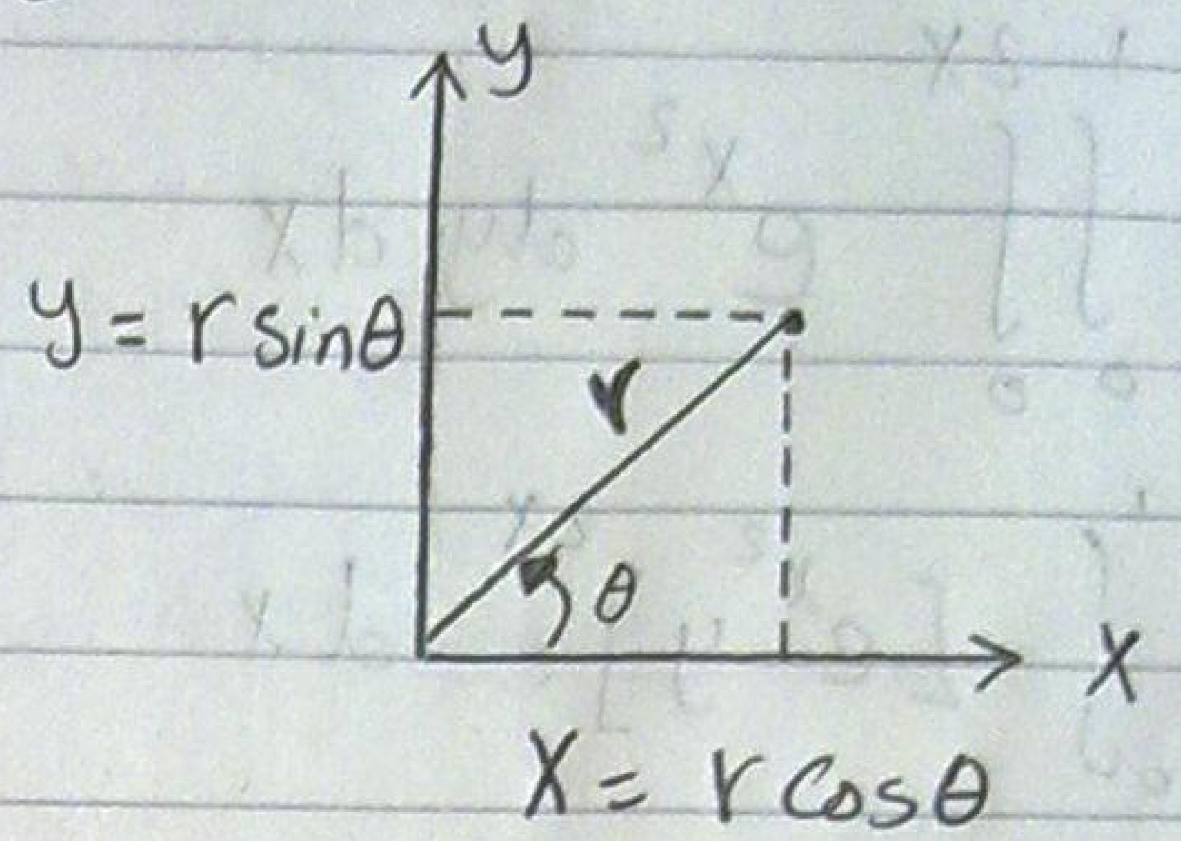
Double Integration in Polar Coordinates

$$(x, y) \longrightarrow (r, \theta) \quad \& \quad d\ dy \longrightarrow r\,dr\,d\theta$$

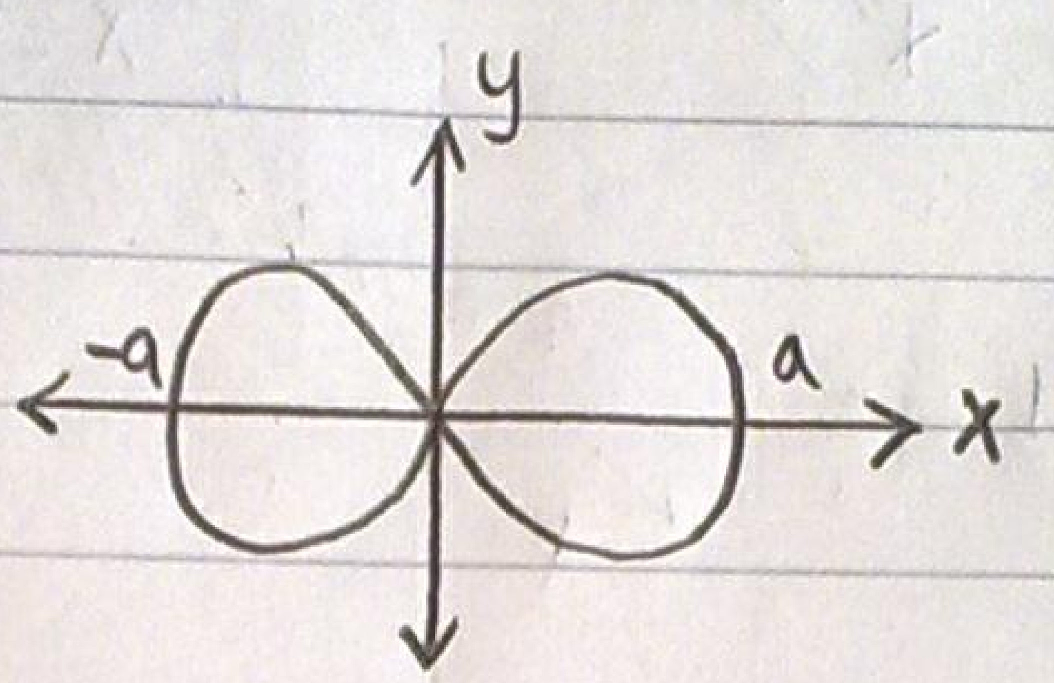
يتم استخدام احداثيات Polar
عند أشكال معينة وهي :-



$$x^2 + y^2 = r^2$$

[1] lemniscate

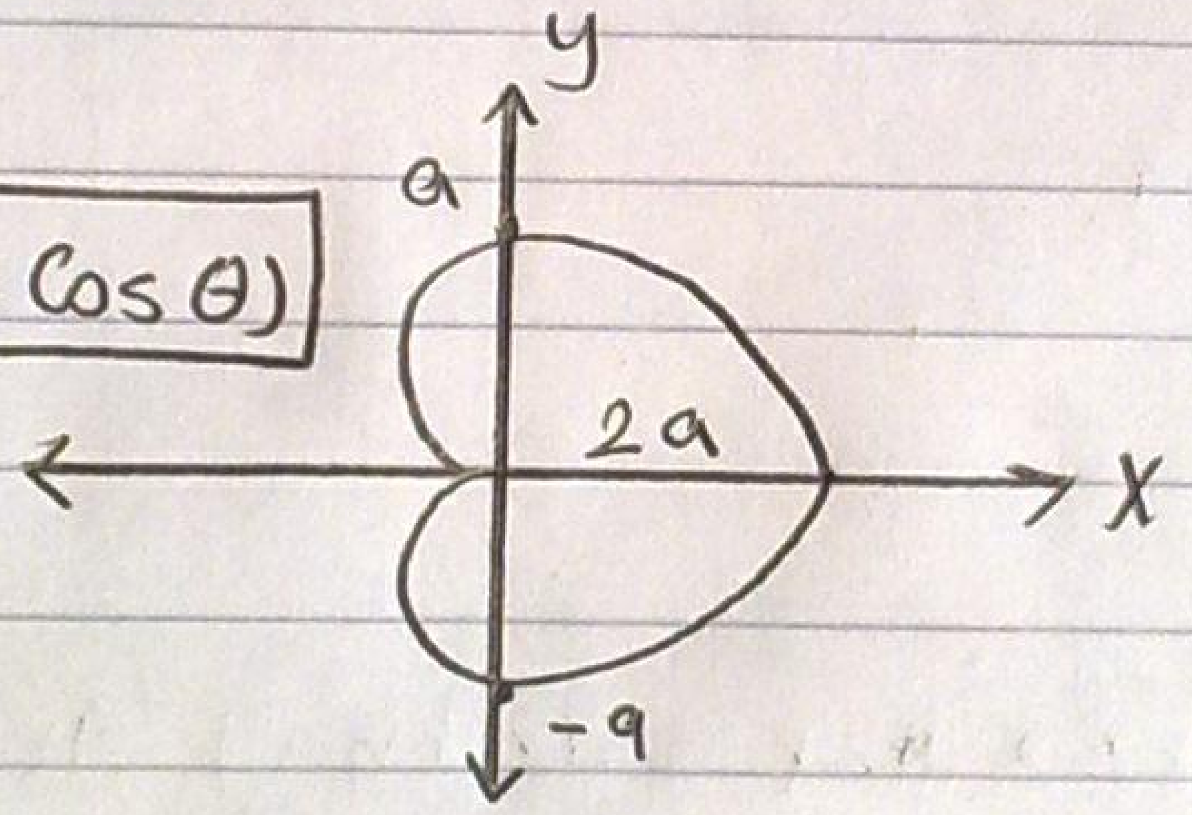
$$\boxed{r^2 = a^2 \cos 2\theta}$$



[2] Cardoid

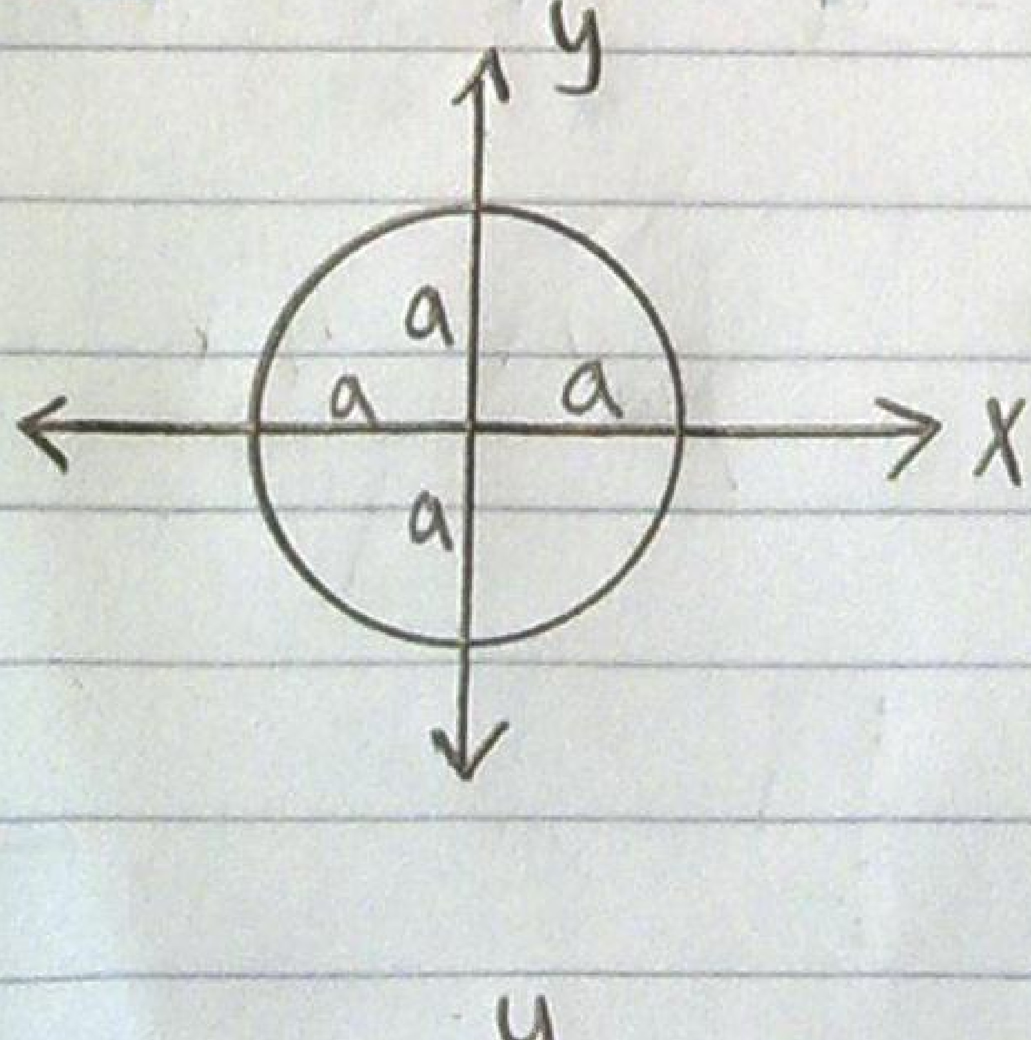
الكاردويد

$$\boxed{r = a(1 + \cos\theta)}$$



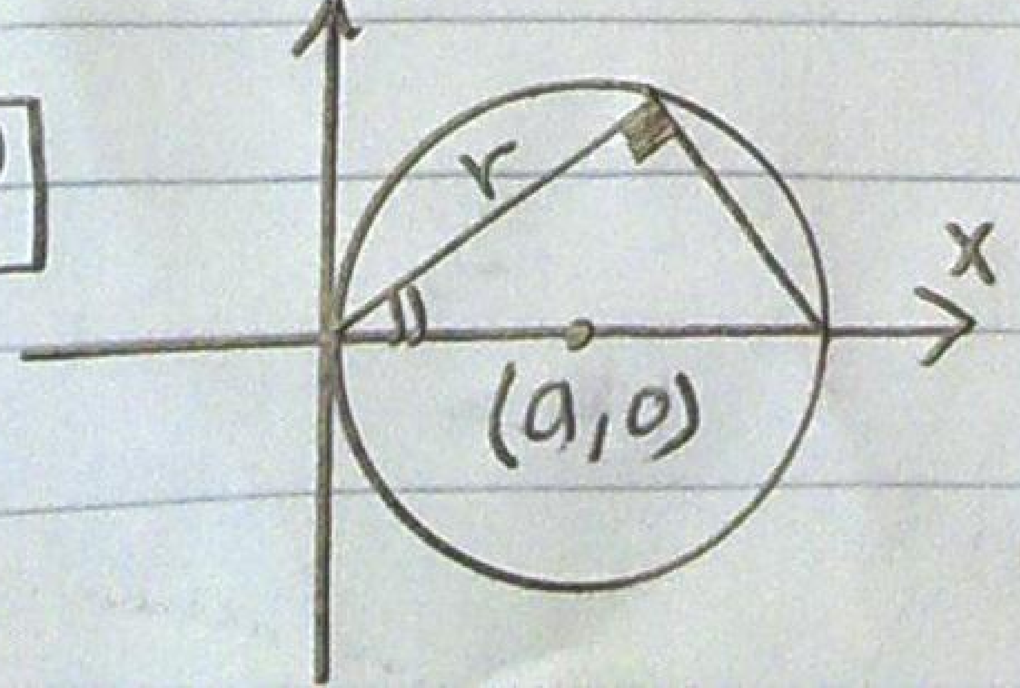
[3] circle

$$\boxed{r = a}$$



[4]

$$\boxed{r = 2a\cos\theta}$$



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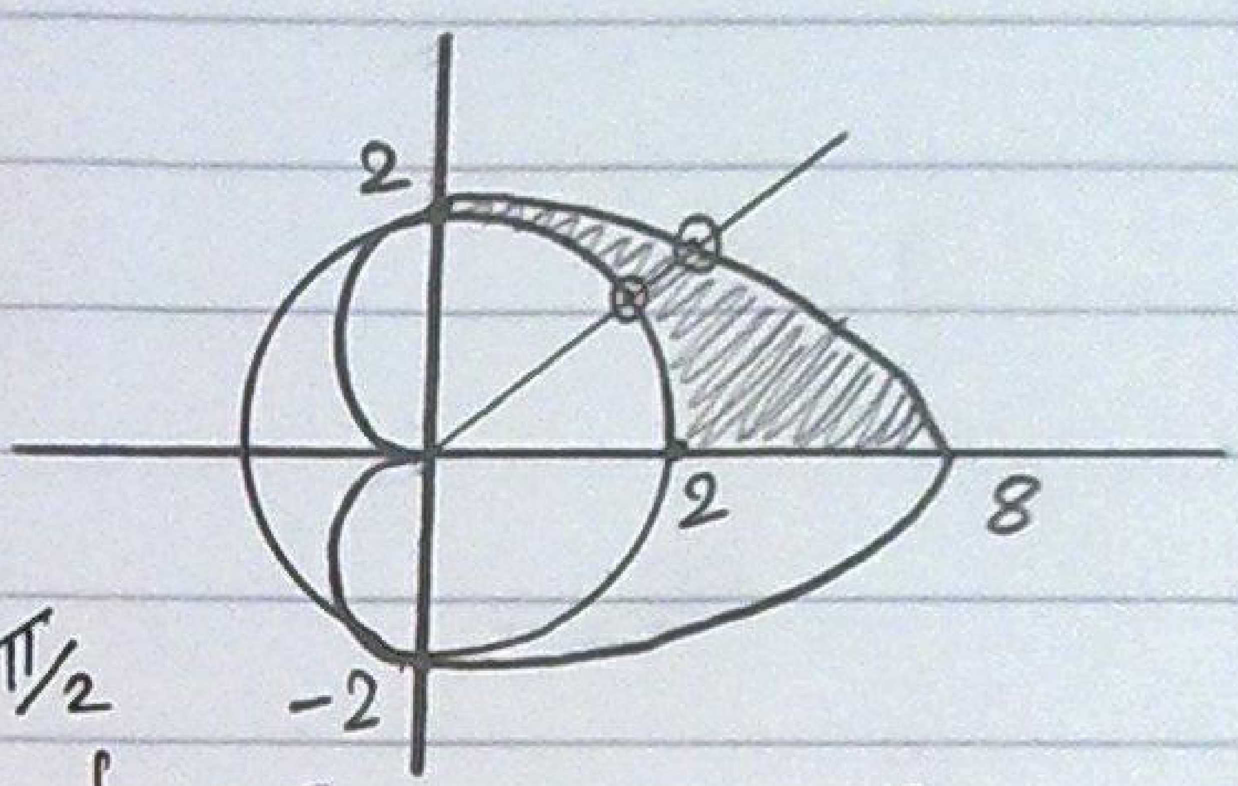
Evaluate $\iint\limits_R \sin\theta \cdot dA$ where R is the region in the first quadrant that is Outside the circle $r=2$ and inside the Cardioid $r=2(1+\cos\theta)$

$$= \int_0^{\pi/2}\int_2^{2(1+\cos\theta)} \sin\theta \; r\,dr\,d\theta$$



$$= \int_0^{\pi/2} \sin\theta \left[\frac{r^2}{2}\right]_2^{2(1+\cos\theta)} = \int_0^{\pi/2} \left[2(1+\cos\theta)^2\sin\theta - 2\sin\theta\right] d\theta$$

$$= \int_0^{\pi/2} \left[\cancel{2\sin\theta} + 2\sin\theta\cos^2\theta + 4\sin\theta\cos\theta - \cancel{2\sin\theta}\right] d\theta$$

$$= \int_0^{\pi/2} (2\sin\theta\cos^2\theta + 2\sin 2\theta)\, d\theta$$

$$= \left[-2\frac{\cos^3\theta}{3} - \frac{2}{2}\cos 2\theta\right]_0^{\pi/2}$$

$$= \left[-\frac{2}{3}\cos^3\theta - \cos 2\theta\right]_0^{\pi/2}$$

$$= \frac{-2}{3}\underbrace{\cos^{\cancel{3}} 90}_{0} - \cos 180 + \frac{2}{3}\cos^3 0 + \cos 0$$

$$= 1 + \frac{2}{3} + 1 = 2\,\frac{2}{3} = \boxed{\frac{8}{3}}$$

ملحوظة

* استبدال المتغيرات ملغية
* الالتزام بما قيل في المحاضرة لأن الكتاب فيه حاجات زيادة